UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET	
COURSE CODE	: BGE 1123
COURSE	: ALGEBRA
SEMESTER/SESSION	: 1-2022/2023
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **6** questions in SECTION A, **3** questions in SECTION B and **2** questions in SECTION C. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 6 PRINTED PAGES INCLUDING COVER PAGE

SECTION A (50 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Given a matrix $Q=\begin{pmatrix} x & 4 & -x \\ 2 & 0 & 5 \\ 0 & -2 & -x \end{pmatrix}$ and determinant of Q is 66,

a) Find the value of x. (3 marks)

b) Hence, find the inverse of Q. (5 marks)

QUESTION 2

Solve the equation $y^2+9y-11=4(3y-5)$ by using completing the square.

(6 marks)

QUESTION 3

a) Given $15\cot\theta=8$, find $\sin\theta$ and $\sec\theta$. (5 marks)

b) Hence, find the area for the above triangle. (2 marks)

QUESTION 4

If $z_1=1+i$ and $z_2=3+2i$, find $\frac{z_1}{z_2}$ (4 marks)

QUESTION 5

Given $f(x)=\frac{x+6}{x+2}$, $x\in\mathbb{R}$, $x\neq 2$ and $g(x)=7-2x^2$, $x\in\mathbb{R}$.

a) Find $f^{-1}(x)$. (4 marks)

b) State the domain and range for both functions, $f(x)$ and $g(x)$. (4 marks)

c) Find the composite function of $fg(x)$ and $gg(-3)$. (6 marks)

QUESTION 6

Given three vectors, $\vec{u} = 5\mathbf{i} + 13\mathbf{j}$, $\vec{v} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $\vec{w} = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$. Find each of the following:

a) $\vec{v} \cdot \vec{w}$. (2 marks)

b) $\vec{u} \cdot (3\vec{u} - 2\vec{v})$. (3 marks)

c) the area of parallelogram formed by $(\vec{u} + \vec{v})$ and $\vec{w}$. (6 marks)

SECTION B (30 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

a) Solve the following trigonometric equation:

$$3\cos 2\theta - 1 = 0.22, \quad 0° \le \theta \le 360°$$ (6 marks)

b) Prove that $(1-\cos\theta)(1+\cos\theta)(1+\cot^2\theta)=1$. (4 marks)

QUESTION 2

a) Use De Moivre's theorem to simplify $(4+4i)^5$. Write the answer in the form of $a+bi$. (Give your answer to the nearest whole number).

(5 marks)

b) Find the value of x and y if $(x-iy)(3+5i)$ is the conjugate of $-6-24i$.

(7 marks)

QUESTION 3

Express $\dfrac{x^4+2x^3-2x^2+4x-1}{x^2+2x-3}$ as a partial fractions. (8 marks)

SECTION C (20 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

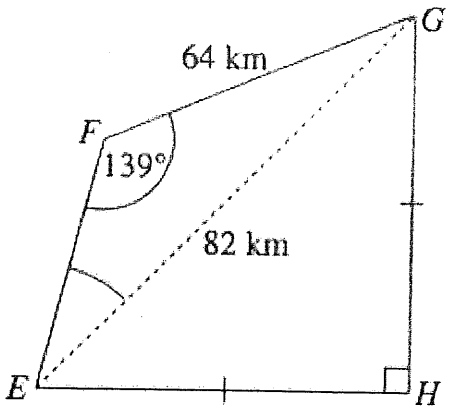
QUESTION 1

A theater has 490 seats. Seats sell for RM100 on the floor, RM80 in the mezzanine, and RM60 in the balcony. The number of seats on the floor equals the number of seats in the mezzanine and balcony. Suppose the theater takes in RM 42,080 from each sold-out event. Find the number of seats on the floor, in the mezzanine and in the balcony by using Gauss Elimination method.

(10 marks)

QUESTION 2

Nastya cycles around a course. The course starts at E, passes through F, G, and H and finishes at E. The distance EH and GH are equal.



NOT TO SCALE

a) What is the length of EF, to the nearest kilometre? (5 marks)

b) What is the total distance that Nastya cycles, to the nearest kilometre?

(5 marks)

----------END OF QUESTION----------

FORMULA

$$A^{-1} = \frac{1}{|A|} adj(A)$$

$$adj(A) = \left[A_{ij}\right]^T$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$H^2 = A^2 + O^2$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$Area = \frac{1}{2} ab\sin C$$

$$r = |z| = \sqrt{\left(a^2 + b^2\right)}$$

$$\theta = Arg(z) = \tan^{-1}\left|\frac{b}{a}\right|$$

$$z = r\left(\cos\theta + i\sin\theta\right)$$

$$z^n = r^n\left(\cos n\theta + i\sin n\theta\right)$$

$$z = re^{i\theta}$$

$$i^2 = -1$$

$$|u| = \sqrt{a^2 + b^2 + c^2}$$

$$\hat{u} = \frac{u}{|u|} = \frac{ai + bj + ck}{\sqrt{a^2 + b^2 + c^2}}$$

$$u \cdot v = a_1a_2 + b_1b_2 + c_1c_2$$

$$\theta = \cos^{-1}\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\right)$$

$$|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\sin\theta$$

$$u \times v = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} i - \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} j + \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} k$$

$$= \left(b_1c_2 - b_2c_1\right)i - \left(a_1c_2 - a_2c_1\right)j + \left(a_1b_2 - a_2b_1\right)k$$